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Situation: Converting Repeating Decimals to Fractions

Prompt: In a professional development project, four middle school teachers met on a monthly basis and shared videotapes of their classrooms. One of the teachers was sharing his 7th grade lesson on converting decimals to fraction beginning with terminating decimals and then continuing to repeating decimals to fractions. He created a worksheet that included the following "rules":

"Converting simple repeating decimals to fractions:

- ❑ Simple repeating decimals will always have "nines" in the denominator
- ❑ The numerator is the repeating pattern
- ❑ However many digits are in the repeating pattern, that is how many 9's are in the denominator
- ❑ Reduce to lowest terms
- ❑ Example: 0.36363636…; numerator = 36, denominator = 99; 36/99 = 4/11"

After sharing this worksheet, another one of the teachers asked, "Does this ALWAYS work?" and another asked, "How does this work? I've never seen this before." And the third asked, "What is a 'simple' repeating decimal and how is different from just a repeating decimal?"

Mathematical Focus 1: *Rational numbers, whether terminating or repeating non-terminating decimals, can be represented as a fraction. On the other hand, non-repeating non-terminating decimals are irrational and can not be represented as a fraction.*

Mathematical Focus 2: *Converting repeating non-terminating decimals into fractions can be done using algebraic processes and equations. For example, if n=0.363636… and 100n=36.3636…, then 100n- n =36.3636…-0.3636… Hence 99n=36, so n= 36/99. But note, that depending on the repeating non-terminating decimal, it will not always result in the denominator being, 9 or 99 or 999, etc. Try n=0.0252525…..*

Situation: Determining the Volume of Cylinder

Prompt: A 7th grade classroom was applying the formula for finding the volume of a cylinder to real situations. The teacher reminded the students of the formula for finding the volume as $V=\pi r^2 h$. She then handed out meter sticks, rulers, and calculators and asked students, in their groups, to find the volume for a cylindrical trash can and the tape in a roll of masking tape.

When the whole class came back together, there was consensus on the volume for the trash can, but when it came to sharing solutions to the masking tape, three separate solution strategies emerged. (The dimensions of the masking tape were: outer diameter of tape was 12 cm, the inner diameter was 8 cm, and the height of the tape was 2 cm.)

Solution 1: Found volume of "big" and subtract volume of "small": $226.08 - 100.48 = 125.60 \text{ cm}^3$

Solution 2: Found the width of the tape (2 cm) and used the formula: $V= 2^2 \times \pi \times 2 = 25.12 \text{ cm}^3$

Solution 3: Found the radius of the outer ring and squared it, found the radius of the inner ring and squared it and subtracted the two. Multiplied this result by π and the height: $V=(6^2 - 4^2) \times \pi \times 2 = 40\pi$

Solution 4: Found the radius of the inner ring and subtracted it from the radius of the outer ring. Squared that result and multiplied it by π and the height: $V= (6 - 4)^2 \times \pi \times 2 = 4\pi$.

Solution 5: Found the circumference of the outer ring and added that to the circumference of the inner ring. Averaged the two circumferences and then multiply it by the width of the tape and then the height: $V= \frac{1}{2} (12\pi + 8\pi) \times 2 \times 2 = 125.60 \text{ cm}^3$

How are Solutions 1,2, 3, and 4 similar and different? What should the teacher say to help students understand why Solution 2 does not work? Is Solution 5 generalizeable or does it just work in this situation?

Mathematical Focus 1: *The relationships among the a) circumference of a circular base b) area of a circular base, and 3) the volume of a cylinder.*

Mathematical Focus 2: *The distributive property is important to understand when comparing and contrasting and confirming that the various methods of solving are indeed appropriate.*

Mathematical Focus 3: *(Possible) What constitutes "exact" values when solving volumes or any problem involving irrational numbers (e.g., π)? And if using approximate values, when does one round during the series of calculations?*